

Module 4

Single-phase AC circuits

Lesson 15

Solution of Current in AC Series and Parallel Circuits

In the last lesson, two points were described:

1. How to solve for the impedance, and current in an ac circuit, consisting of single element, R / L / C?
2. How to solve for the impedance, and current in an ac circuit, consisting of two elements, R and L / C, in series, and then draw complete phasor diagram?

In this lesson, the solution of currents in simple circuits, consisting of resistance R, inductance L and/or capacitance C connected in series, fed from single phase ac supply, is presented. Then, the circuit with all above components in parallel is taken up. The process of drawing complete phasor diagram with current(s) and voltage drops in the different components is described. The computation of total power and also power consumed in the different components, along with power factor, is explained. One example of series circuit are presented in detail, while the example of parallel circuit will be taken up in the next lesson.

Keywords: Series and parallel circuits, impedance, admittance, power, power factor.

After going through this lesson, the students will be able to answer the following questions;

1. How to compute the total reactance and impedance / admittance, of the series and parallel circuits, fed from single phase ac supply?
2. How to compute the different currents and also voltage drops in the components, both in magnitude and phase, of the circuit?
3. How to draw the complete phasor diagram, showing the currents and voltage drops?
4. How to compute the total power and also power consumed in the different components, along with power factor?

Solution of Current in R-L-C Series Circuit

Series (R-L-C) circuit

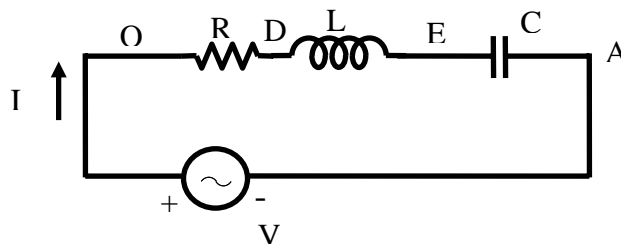


Fig. 15.1 (a) Circuit diagram

The voltage balance equation for the circuit with R, L and C in series (Fig. 15.1a), is

$$v = Ri + L \frac{di}{dt} + \frac{1}{C} \int i dt = \sqrt{2} V \sin \omega t$$

The current, i is of the form,

$$i = \sqrt{2} I \sin(\omega t \pm \phi)$$

As described in the previous lesson (#14) on series (R-L) circuit, the current in steady state is sinusoidal in nature. The procedure given here, in brief, is followed to determine the form of current. If the expression for $i = \sqrt{2} I \sin(\omega t - \phi)$ is substituted in the voltage equation, the equation shown here is obtained, with the sides (LHS & RHS) interchanged.

$$\begin{aligned} R \cdot \sqrt{2} I \sin(\omega t - \phi) + \omega L \cdot \sqrt{2} I \cos(\omega t - \phi) - (1/\omega C) \cdot \sqrt{2} I \cos(\omega t - \phi) \\ = \sqrt{2} V \sin \omega t \end{aligned}$$

$$\text{or } R \cdot \sqrt{2} I \sin(\omega t - \phi) + [\omega L - (1/\omega C)] \cdot \sqrt{2} I \cos(\omega t - \phi) = \sqrt{2} V \sin \omega t$$

The steps to be followed to find the magnitude and phase angle of the current I , are same as described there (#14).

$$\text{So, the phase angle is } \phi = \tan^{-1}[\omega L - (1/\omega C)]/R$$

$$\text{and the magnitude of the current is } I = V/Z$$

$$\text{where the impedance of the series circuit is } Z = \sqrt{R^2 + [\omega L - (1/\omega C)]^2}$$

Alternatively, the steps to find the rms value of the current I , using complex form of impedance, are given here.

The impedance of the circuit is

$$Z \angle \pm \phi = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

where,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - (1/\omega C))^2}, \text{ and}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega L - (1/\omega C)}{R}\right)$$

$$\bar{I} \angle \mp \phi = \frac{\bar{V} \angle 0^\circ}{Z \angle \pm \phi} = \frac{V + j0}{R + j(X_L - X_C)} = \frac{V + j0}{R + j(\omega L - (1/\omega C))}$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\text{Two cases are: (a) Inductive } \left(\omega L > \frac{1}{\omega C}\right), \text{ and (b) Capacitive } \left(\omega L < \frac{1}{\omega C}\right).$$

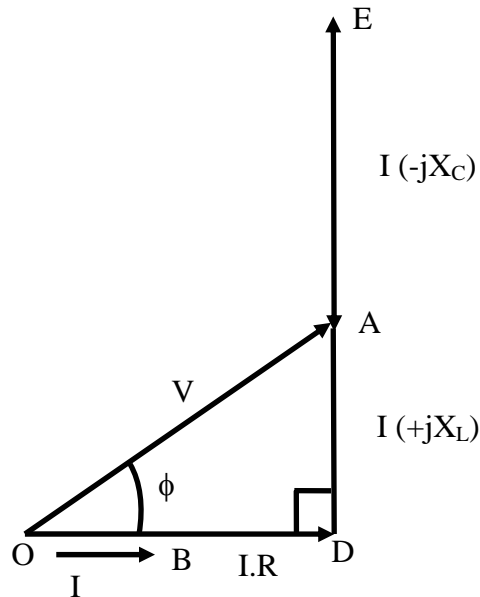
(a) Inductive

In this case, the circuit is inductive, as total reactance $(\omega L - (1/\omega C))$ is positive, under the condition $(\omega L > (1/\omega C))$. The current lags the voltage by ϕ (taken as positive), with the voltage phasor taken as reference. The power factor (lagging) is less than 1 (one), as $0^\circ \leq \phi \leq 90^\circ$. The complete phasor diagram, with the voltage drops across the

components and input voltage (OA), and also current (OB), is shown in Fig. 15.1b. The voltage phasor is taken as reference, in all cases. It may be observed that

$$V_{OC} (= iR) + V_{CD} [= i(jX_L)] + V_{DA} [= -i(jX_C)] = V_{OA} (= iZ)$$

using the Kirchoff's second law relating to the voltage in a closed loop. The phasor diagram can also be drawn with the current phasor as reference, as will be shown in the example given here. The expression for the average power is $V I \cos \phi = I^2 R$. The power is only consumed in the resistance, R , but not in inductance/capacitance (L/C), in all three cases.



Inductive ($X_L > X_C$)

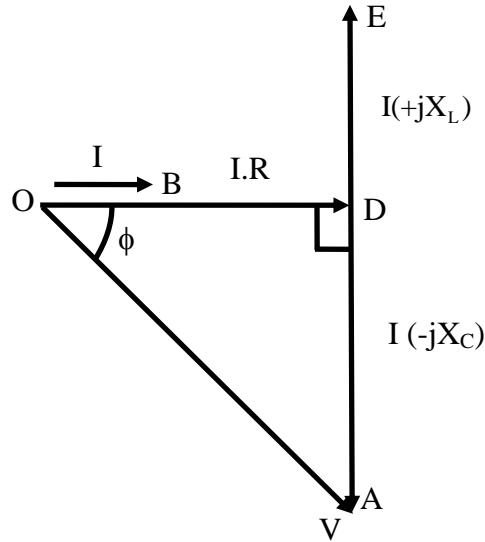
Fig 15.1 (b) Phasor diagram

In this case, the circuit is inductive, as total reactance $(\omega L - (1/\omega C))$ is positive, under the condition $(\omega L > (1/\omega C))$. The current lags the voltage by ϕ (positive). The power factor (lagging) is less than 1 (one), as $0^\circ \leq \phi \leq 90^\circ$. The complete phasor diagram, with the voltage drops across the components and input voltage (OA), and also current (OB), is shown in Fig. 15.1b. The voltage phasor is taken as reference, in all cases. It may be observed that

$$V_{OC} (= iR) + V_{CD} [= i(jX_L)] + V_{DA} [= -i(jX_C)] = V_{OA} (= iZ)$$

using the Kirchoff's second law relating to the voltage in a closed loop. The phasor diagram can also be drawn with the current phasor as reference, as will be shown in the example given here. The expression for the average power is $V I \cos \phi = I^2 R$. The power is only consumed in the resistance, R , but not in inductance/capacitance (L/C), in all three cases.

(b) Capacitive

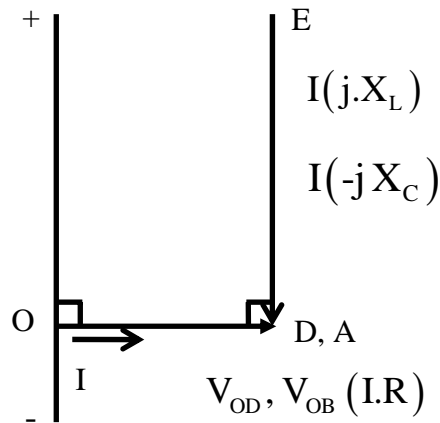


Capacitive ($X_L < X_C$)

Fig 15.1 (c) Phasor diagram

The circuit is now capacitive, as total reactance $(\omega L - (1/\omega C))$ is negative, under the condition $(\omega L < (1/\omega C))$. The current leads the voltage by ϕ , which is negative as per convention described in the previous lesson. The voltage phasor is taken as reference here. The complete phasor diagram, with the voltage drops across the components and input voltage, and also current, is shown in Fig. 15.1c. The power factor (leading) is less than 1 (one), as $0^\circ \leq \phi \leq 90^\circ$, ϕ being negative. The expression for the average power remains same as above.

The third case is resistive, as total reactance $(\omega L - 1/\omega C)$ is zero (0), under the condition $(\omega L = 1/\omega C)$. The impedance is $Z \angle 0^\circ = R + j0$. The current is now at unity power factor ($\phi = 0^\circ$), i.e. the current and the voltage are in phase. The complete phasor diagram, with the voltage drops across the components and input (supply) voltage, and also current, is shown in Fig. 15.1d. This condition can be termed as ‘resonance’ in the series circuit, which is described in detail in lesson #17. The magnitude of the impedance in the circuit is minimum under this condition, with the magnitude of the current being maximum. One more point to be noted here is that the voltage drops in the inductance, L and also in the capacitance, C, is much larger in magnitude than the supply voltage, which is same as the voltage drop in the resistance, R. The phasor diagram has been drawn approximately to scale.



Resistive ($X_L = X_C$)

Fig. 15.1 (d) Phasor diagram

It may be observed here that two cases of series (R-L & R-C) circuits, as discussed in the previous lesson, are obtained in the following way. The first one (inductive) is that of (a), with C very large, i.e. $1/\omega C \approx 0$, which means that C is not there. The second one (capacitive) is that of (b), with L not being there (L or $\omega L = 0$).

Example 15.1

A resistance, R is connected in series with an iron-cored choke coil (r in series with L). The circuit (Fig. 15.2a) draws a current of 5 A at 240 V, 50 Hz. The voltages across the resistance and the coil are 120 V and 200 V respectively. Calculate,

- the resistance, reactance and impedance of the coil,
- the power absorbed by the coil, and
- the power factor (pf) of the input current.

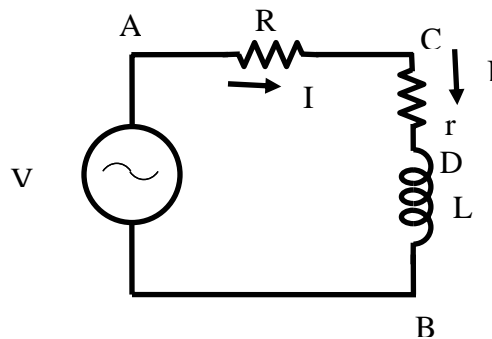


Fig. 15.2 (a) Circuit diagram

Solution

$$I(OB) = 5 \text{ A} \quad V_s(OA) = 240 \text{ V} \quad f = 50 \text{ Hz} \quad \omega = 2\pi f$$

$$\text{The voltage drop across the resistance } V_1(OC) = I \cdot R = 120 \text{ V}$$

The resistance, $R = V_1 / I = 120 / 5 = 24 \Omega$

The voltage drop across the coil $V_2 (CA) = I \cdot Z_L = 200 \text{ V}$

The impedance of the coil, $Z_L = \sqrt{r^2 + X_L^2} = V_2 / I = 200 / 5 = 40 \Omega$

From the phasor diagram (Fig. 15.2b),

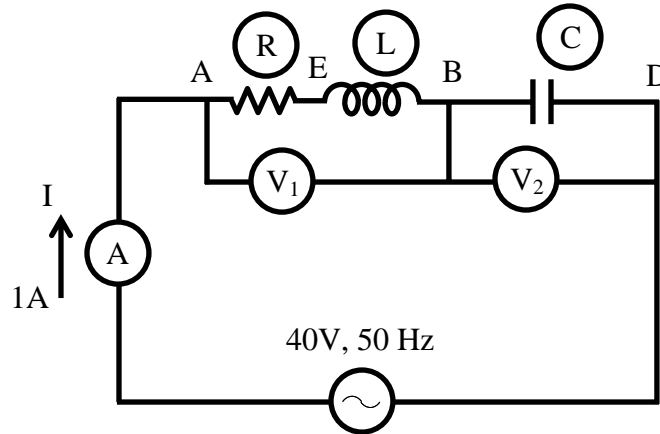


Fig. 15.2(b): Phasor Diagram

$$\cos \phi = \cos \angle AOC = \frac{OA^2 + OC^2 - CA^2}{2 \cdot OA \cdot OC} = \frac{(120)^2 + (240)^2 - (200)^2}{2 \times 120 \times 240} = \frac{32,000}{57,600} = 0.556$$

The power factor (pf) of the input current = $\cos \phi = 0.556$ (lag)

The phase angle of the total impedance, $\phi = \cos^{-1}(0.556) = 56.25^\circ$

Input voltage, $V_s (OA) = I \cdot Z = 240 \text{ V}$

The total impedance of the circuit, $Z = \sqrt{(R+r)^2 + X_L^2} = V_s / I = 240 / 5 = 48 \Omega$

$$Z \angle \phi = (R+r) + jX_L = 48 \angle 56.25^\circ = (26.67 + j39.91) \Omega$$

The total resistance of the circuit, $R+r = 24 + r = 26.67 \Omega$

The resistance of the coil, $r = 26.67 - 24.0 = 2.67 \Omega$

The reactance of the coil, $X_L = \omega L = 2 \pi f L = 39.9 \Omega$

$$\text{The inductance of the coil, } L = \frac{X_L}{2 \pi f} = \frac{39.9}{2 \pi \times 50} = 0.127 \text{ H} = 127 \cdot 10^{-3} = 127 \text{ mH}$$

The phase angle of the coil,

$$\phi_L = \cos^{-1}(r/Z_L) = \cos^{-1}(2.67/40.0) = \cos^{-1}(0.067) = 86.17^\circ$$

$$Z_L \angle \phi_L = r + jX_L = (2.67 + j39.9) = 40 \angle 86.17^\circ \Omega$$

The power factor (pf) of the coil, $\cos \phi = 0.067$ (lag)

$$\text{The copper loss in the coil} = I^2 r = 5^2 \times 2.67 = 66.75 \text{ W}$$

Example 15.2

An inductive coil, having resistance of $8\ \Omega$ and inductance of 80 mH , is connected in series with a capacitance of $100\ \mu\text{F}$ across 150 V , 50 Hz supply (Fig. 15.3a). Calculate, (a) the current, (b) the power factor, and (c) the voltages drops in the coil and capacitance respectively.

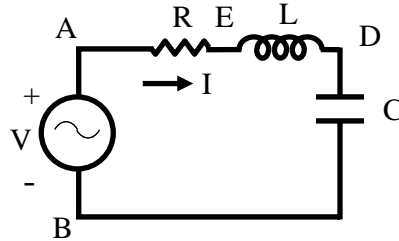


Fig. 15.3 (a) Circuit diagram

Solution

$$f = 50\text{ Hz}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16\text{ rad/s}$$

$$L = 80\text{ mH} = 80 \cdot 10^{-3} = 0.08\text{ H}$$

$$X_L = \omega L = 314.16 \times 0.08 = 25.13\ \Omega$$

$$C = 100\ \mu\text{F} = 100 \cdot 10^{-6}\text{ F}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314.16 \times 100 \cdot 10^{-6}} = 31.83\ \Omega$$

$$R = 8\ \Omega$$

$$V_s\text{ (OA)} = 150\text{ V}$$

The impedance of the coil, $Z_L \angle \phi_L = R + jX_L = (8.0 + j25.13) = 26.375 \angle 72.34^\circ\ \Omega$

The total impedance of the circuit,

$$Z \angle -\phi = R + j(X_L - X_C) = 8.0 + j(25.13 - 31.83) = (8.0 - j6.7) \\ = 10.435 \angle -39.95^\circ\ \Omega$$

The current drawn from the supply,

$$I \angle \phi = \frac{V \angle 0^\circ}{Z \angle -\phi} = \left(\frac{150}{10.435} \right) \angle 39.95^\circ = 14.375 \angle 39.95^\circ\text{ A} = (11.02 + j9.26)\text{ A}$$

The current is, $I = 14.375\text{ A}$

The power factor (pf) = $\cos \phi = \cos 39.95^\circ = 0.767$ (lead)

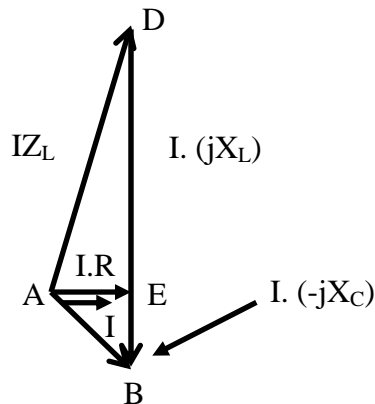


Fig. 15.3 (b) Phasor diagram

Please note that the current phasor is taken as reference in the phasor diagram (Fig. 15.3b) and also here. The voltage drop in the coil is,

$$V_1 \angle \theta_1 = I \angle 0^\circ \cdot Z_L \angle \phi_L = (14.375 \times 26.375) \angle 72.34^\circ = 379.14 \angle 72.34^\circ \text{ V} \\ = (115.1 + j 361.24) \text{ V}$$

The voltage drop in the capacitance in,

$$V_2 \angle \theta_2 = I \angle 0^\circ \cdot Z_C \angle -\phi_C = (14.475 \times 31.83) \angle -90.0^\circ = 457.58 \angle -90.0^\circ \text{ V} \\ = -j 457.58 \text{ V}$$

Solution of Current in Parallel Circuit

Parallel circuit

The circuit with all three elements, R, L & C connected in parallel (Fig. 15.4a), is fed to the ac supply. The current from the supply can be computed by various methods, of which two are described here.

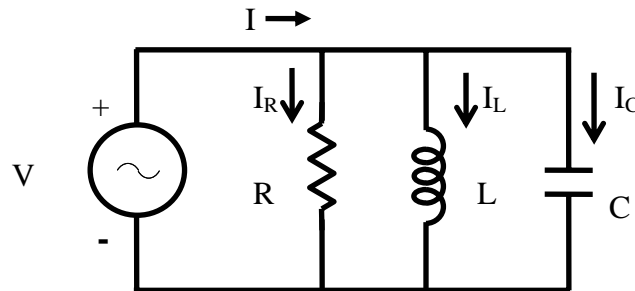


Fig. 15.4 (a) Circuit diagram.

First method

The current in three branches are first computed and the total current drawn from the supply is the phasor sum of all three branch currents, by using Kirchhoff's first law related to the currents at the node. The voltage phasor (V) is taken as reference.

All currents, i.e. three branch currents and total current, in steady state, are sinusoidal in nature, as the input (supply voltage is sinusoidal of the form,

$$v = \sqrt{2} V \sin \omega t$$

Three branch currents are obtained by the procedure given in brief.

$$v = R \cdot i_R, \text{ or } i_R = v / R = \sqrt{2} (V / R) \sin \omega t = \sqrt{2} I_R \sin \omega t,$$

$$\text{where, } |I_R| = |(V / R)|$$

$$\text{Similarly, } v = L \frac{di_L}{dt}$$

So, i_L is,

$$i_L = (1/L) \int v dt = (1/L) \int \sqrt{2} V (\sin \omega t) dt = -\sqrt{2} [V / (\omega L)] \cos \omega t = -\sqrt{2} I_L \cos \omega t \\ = \sqrt{2} I_L \sin (\omega t - 90^\circ)$$

where, $|I_L| = |(V / X_L)|$ with $X_L = \omega L$

$v = (1/C) \int i_C dt$, from which i_C is obtained as,

$$\begin{aligned} i_C &= C \frac{dv}{dt} = C \frac{d}{dt} (\sqrt{2} V \sin \omega t) = \sqrt{2} (V \cdot \omega C) \cos \omega t = \sqrt{2} I_C \cos \omega t \\ &= \sqrt{2} I_C \sin(\omega t + 90^\circ) \end{aligned}$$

where, $|I_C| = |(V / X_C)|$ with $X_C = (1/\omega C)$

Total (supply) current, i is

$$\begin{aligned} i &= i_R + i_L + i_C = \sqrt{2} I_R \sin \omega t - \sqrt{2} I_L \cos \omega t + \sqrt{2} I_C \cos \omega t \\ &= \sqrt{2} I_R \sin \omega t - \sqrt{2} (I_L - I_C) \cos \omega t = \sqrt{2} I \sin(\omega t \mp \phi) \end{aligned}$$

The two equations given here are obtained by expanding the trigonometric form appearing in the last term on RHS, into components of $\cos \omega t$ and $\sin \omega t$, and then equating the components of $\cos \omega t$ and $\sin \omega t$ from the last term and last but one (previous).

$$I \cos \phi = I_R \text{ and } I \sin \phi = (I_L - I_C)$$

From these equations, the magnitude and phase angle of the total (supply) current are,

$$\begin{aligned} |I| &= \sqrt{(I_R)^2 + (I_L - I_C)^2} = |V| \cdot \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2} \\ &= |V| \cdot \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L} - \omega C\right)^2} = |V| \cdot |Y| \\ \phi &= \tan^{-1} \left(\frac{I_L - I_C}{I_R} \right) = \tan^{-1} \left(\frac{(1/X_L) - (1/X_C)}{(1/R)} \right) = \tan^{-1} \left[R \cdot \left(\frac{1}{X_L} - \frac{1}{X_C} \right) \right] \\ &= \tan^{-1} \left[R \cdot \left(\frac{1}{\omega L} - \omega C \right) \right] \end{aligned}$$

where, the magnitude of the term (admittance of the circuit) is,

$$|Y| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

Please note that the admittance, which is reciprocal of impedance, is a complex quantity. The angle of admittance or impedance, is same as the phase angle, ϕ of the current I , with the input (supply) voltage taken as reference phasor, as given earlier.

Alternatively, the steps required to find the rms values of three branch currents and the total (supply) current, using complex form of impedance, are given here.

Three branch currents are

$$I_R \angle 0^\circ = I_R = \frac{V}{R}; \quad I_L \angle -90^\circ = -j I_L = \frac{V}{j X_L} = \frac{V}{j \omega L} = -j \frac{V}{\omega L}$$

$$I_C \angle +90^\circ = j I_C = \frac{V}{-j X_C} = \frac{V}{-j (1/\omega C)} = j \omega C V$$

Of the three branches, the first one consists of resistance only, the current, I_R is in phase with the voltage (V). In the second branch, the current, I_L lags the voltage by 90° , as there is inductance only, while in the third one having capacitance only, the current, I_C leads the voltage 90° . All these cases have been presented in the previous lesson.

The total current is

$$I \angle \pm \phi = I_R + j(I_C - I_L) = V \left[\frac{1}{R} + j \left(\omega C - \frac{1}{\omega L} \right) \right]$$

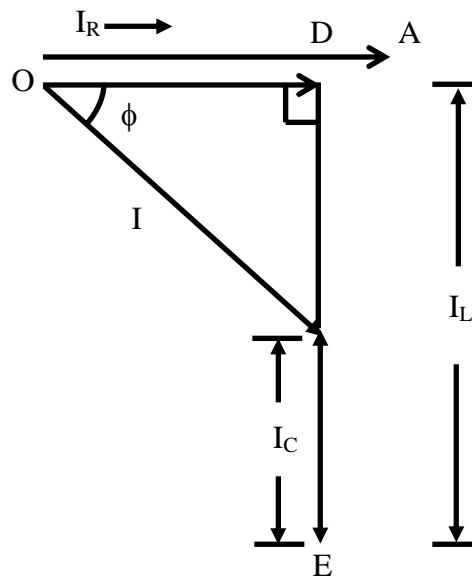
where,

$$I = \sqrt{I_R^2 + (I_C - I_L)^2} = V \sqrt{\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L} \right)^2 \right]}, \text{ and}$$

$$\phi = \tan^{-1} \left(\frac{I_C - I_L}{I_R} \right) = \tan^{-1} \left[R \left(\omega C - \frac{1}{\omega L} \right) \right]$$

The two cases are as described earlier in series circuit.

(a) Inductive



Inductive ($I_L > I_C$)

Fig. 15.4 (b) Phasor diagram

In this case, the circuit being inductive, the current lags the voltage by ϕ (positive), as $I_L > I_C$, i.e. $1/\omega L > \omega C$, or $\omega L < 1/\omega C$. This condition is in contrast to that derived in the case of series circuit earlier. The power factor is less than 1 (one). The complete phasor diagram, with the three branch currents along with total current, and also the voltage, is shown in Fig. 15.4b. The voltage phasor is taken as reference in all cases. It may be observed there that

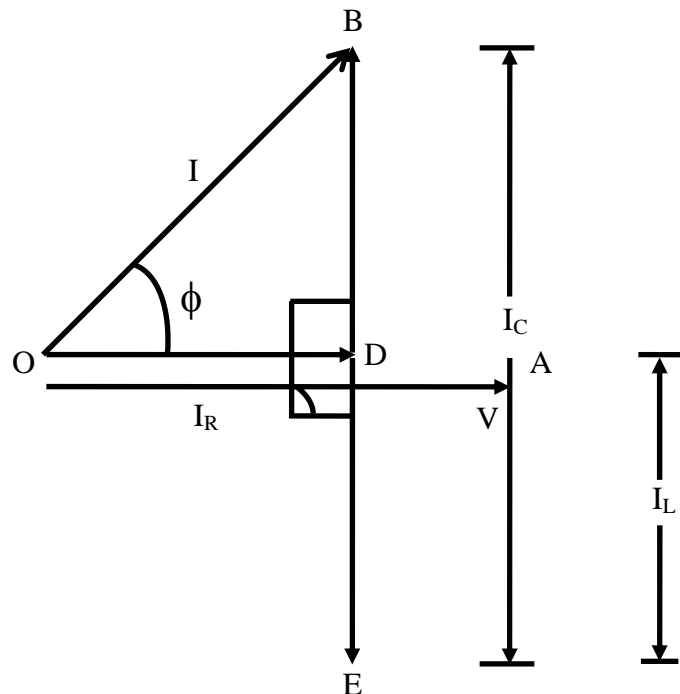
$$I_R(OD) + I_L(DC) + I_C(CB) = I(OB)$$

The Kirchoff's first law related to the currents at the node is applied, as stated above. The expression for the average power is $V I \cos \phi = I_R^2 R = V^2 / R$. The power is only consumed in the resistance, R, but not in inductance/capacitance (L/C), in all three cases.

(b) Capacitive

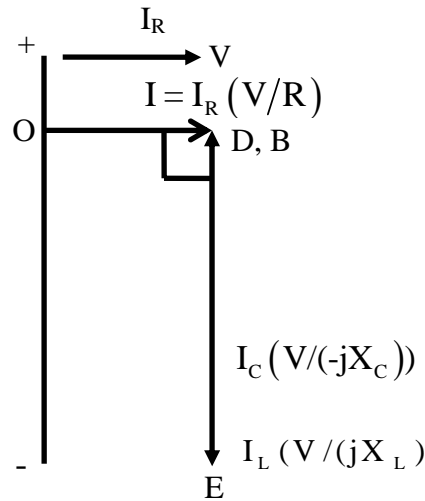
The circuit is capacitive, as $I_C > I_L$, i.e. $\omega C > 1/\omega L$, or $\omega L > 1/\omega C$. The current leads the voltage by ϕ (ϕ being negative), with the power factor less than 1 (one). The complete phasor diagram, with the three branch currents along with total current, and also the voltage, is shown in Fig. 15.4c.

The third case is resistive, as $|I_L| = |I_C|$, i.e. $1/\omega L = \omega C$ or $\omega L = 1/\omega C$. This is the same condition, as obtained in the case of series circuit. It may be noted that two currents, I_L and I_C , are equal in magnitude as shown, but opposite in sign (phase difference being 180°), and the sum of these currents ($I_L + I_C$) is zero (0). The total current is in phase with the voltage ($\phi = 0^\circ$), with $|I| = |I_R|$, the power factor being unity. The complete phasor diagram, with the three branch currents along with total current, and also the voltage, is shown in Fig. 15.4d. This condition can be termed as 'resonance' in the parallel circuit, which is described in detail in lesson #17. The magnitude of the impedance in the circuit is maximum (i.e., the magnitude of the admittance is minimum) under this condition, with the magnitude of the total (supply) current being minimum.



Capacitive ($I_L < I_C$)
Fig. 15.4 (c) Phasor diagram

The circuit with two elements, say R & L, can be solved, or derived with C being large ($I_C = 0$ or $1/\omega C = 0$).



Resistive ($I_L = I_C$)

Fig. 15.4 (d) Phasor Diagram

Second method

Before going into the details of this method, the term, Admittance must be explained. In the case of two resistance connected in series, the equivalent resistance is the sum of two resistances, the resistance being scalar (positive). If two impedances are connected in series, the equivalent impedance is the sum of two impedances, all impedances being complex. Please note that the two terms, real and imaginary, of two impedances and also the equivalent one, may be positive or negative. This was explained in lesson no. 12.

If two resistances are connected in parallel, the inverse of the equivalent resistance is the sum of the inverse of the two resistances. If two impedances are connected in parallel, the inverse of the equivalent impedance is the sum of the inverse of the two impedances. The inverse or reciprocal of the impedance is termed 'Admittance', which is complex. Mathematically, this is expressed as

$$Y = \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = Y_1 + Y_2$$

As admittance (Y) is complex, its real and imaginary parts are called conductance (G) and susceptance (B) respectively. So, $Y = G + jB$. If impedance, $Z \angle \phi = R + jX$ with X being positive, then the admittance is

$$\begin{aligned} Y \angle -\phi &= \frac{1}{Z \angle 0^\circ} = \frac{1}{R + jX} = \frac{R - jX}{(R + jX)(R - jX)} = \frac{R - jX}{R^2 + X^2} \\ &= \frac{R}{R^2 + X^2} - j \frac{X}{R^2 + X^2} = G - jB \end{aligned}$$

where,

$$G = \frac{R}{R^2 + X^2}; \quad B = \frac{X}{R^2 + X^2}$$

Please note the way in which the result of the division of two complex quantities is obtained. Both the numerator and the denominator are multiplied by the complex conjugate of the denominator, so as to make the denominator a real quantity. This has also been explained in lesson no. 12.

The magnitude and phase angle of Z and Y are

$$Z = \sqrt{R^2 + X^2}; \quad \phi = \tan^{-1}(X/R), \text{ and}$$

$$Y = \sqrt{G^2 + B^2} = \frac{1}{\sqrt{R^2 + X^2}}; \quad \phi = \tan^{-1}\left(\frac{B}{G}\right) = \tan^{-1}\left(\frac{X}{R}\right)$$

To obtain the current in the circuit (Fig. 15.4a), the steps are given here.

The admittances of the three branches are

$$Y_1 \angle 0^\circ = \frac{1}{Z_1} = \frac{1}{R}; \quad Y_2 \angle -90^\circ = \frac{1}{Z_2} = \frac{1}{jX_L} = -j\frac{1}{\omega L}$$

$$Y_3 \angle 90^\circ = \frac{1}{Z_3} = \frac{1}{-jX_C} = j\omega C$$

The total admittance, obtained by the phasor sum of the three branch admittances, is

$$Y \angle \pm \phi = Y_1 + Y_2 + Y_3 = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = G + jB$$

where,

$$Y = \sqrt{\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]}; \quad \phi = \tan^{-1}\left[R\left(\omega C - \frac{1}{\omega L}\right)\right], \text{ and}$$

$$G = 1/R; \quad B = \omega C - 1/\omega L$$

The total impedance of the circuit is

$$Z \angle \mp \phi = \frac{1}{Y \angle \pm \phi} = \frac{1}{G + jB} = \frac{G}{G^2 + B^2} - j\frac{B}{G^2 + B^2}$$

The total current in the circuit is obtained as

$$I \angle \pm \phi = \frac{V \angle 0^\circ}{Z \angle \mp \phi} = V \angle 0^\circ \cdot Y \angle \pm \phi = (VY) \angle \pm \phi$$

where the magnitude of current is $I = V \cdot Y = V/Z$

The current is the same as obtained earlier, with the value of Y substituted in the above equation.

This is best illustrated with an example, which is described in the next lesson.

The solution of the current in the series-parallel circuits will also be discussed there, along with some examples.

Problems

- 15.1 Calculate the current and power factor (lagging / leading) for the following circuits (Fig. 15.5a-d), fed from an ac supply of 200 V. Also draw the phasor diagram in all cases.

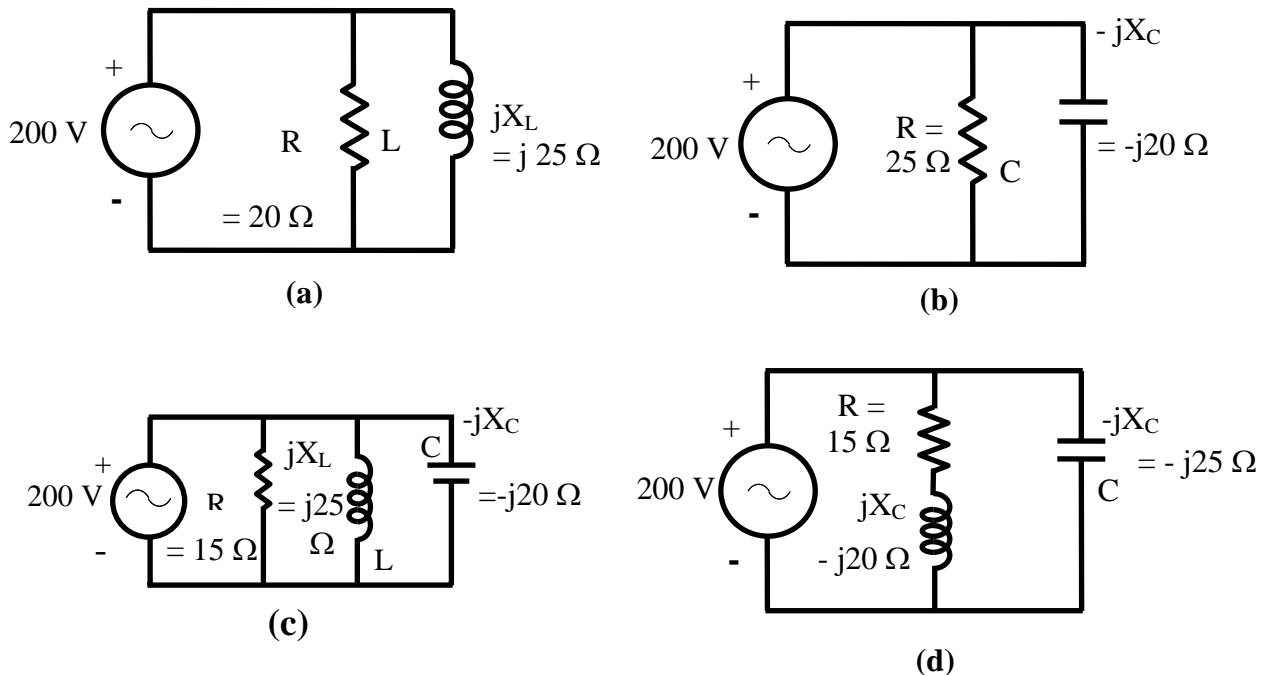


Fig. 15.5

- 15.2 A voltage of 200 V is applied to a pure resistor (R), a pure capacitor, C and a lossy inductor coil, all of them connected in parallel. The total current is 2.4 A, while the component currents are 1.5, 2.0 and 1.2 A respectively. Find the total power factor and also the power factor of the coil. Draw the phasor diagram.
- 15.3 A 200 V, 50Hz supply is connected to a lamp having a rating of 100 V, 200 W, in series with a pure inductance, L, such that the total power consumed is the same, i.e. 200W. Find the value of L.
A capacitance, C is now connected across the supply. Find value of C, to bring the supply power factor to unity (1.0). Draw the phasor diagram in the second case.

- 1.(a) Find the value of the load resistance (R_L) to be connected in series with a real voltage source ($V_S + R_S$ in series), such that maximum power is transferred from the above source to the load resistance.
- (b) Find the voltage across 8Ω resistance in the circuit shown in Fig. 1(b).
- 2.(a) Find the Thevenin's equivalent circuit (draw the ckt.) between the terminals A + B, of the circuit shown in Fig. 2(a).

- (b) A circuit shown in Fig. 2(b) is supplied at 40V, 50Hz. The two voltages V_1 and V_2 (magnitude only) is measured as 60V and 25V respectively. If the current, I is measured as 1A, find the values of R , L and C . Also find the power factor of the circuit (R-L-C). Draw the complete phasor diagram.
- 3.(a) Find the line current, power factor, and active (real) power drawn from 3-phase, 100V, 50Hz, balanced supply in the circuit shown in Fig. 3(a).
- (b) In the circuit shown in Fig. 3(b), the switch, S is put in position 1 at $t = 0$. Find $i_c(t)$, $t > 0$, if $v_c(0^-) = 6V$. After the circuit reaches steady state, the switch, S is brought to position 2, at $t = T_1$. Find $i_c(t)$, $t > T_1$. Sketch the above waveform.
- 4.(a) Find the average and rms values of the periodic waveform shown in Fig. 4(a).
- (b) A coil of 1mH having a series resistance of 1Ω is connected in parallel with a capacitor, C and the combination is fed from 100 mV (0.1V), 1 kHz supply (source) having an internal resistance of 10Ω . If the circuit draws power at unity power factor (upf), determine the value of the capacitor, quality factor of the coil, and power drawn by the circuit. Also draw the phasor diagram.

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- Fig. 15.1 (a) Circuit diagram (R-L-C in series)
(b) Phasor diagram – Circuit is inductive ($X_L > X_C$)
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(d) Phasor diagram – Resistive ($X_L = X_C$)

Fig. 15.2 (a) Circuit diagram (Ex. 15.1), (b) Phasor diagram

Fig. 15.3 (a) Circuit diagram (Ex. 15.2), (b) Phasor diagram

- Fig. 15.4 (a) Circuit diagram (R-L-C in parallel)
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